Microlocal and Global Analysis, Interactions with Geometry

12-16 February 2024, Potsdam

Main organiser: Sylvie Paycha (Potsdam)

Coorganisers:

Chand Devchand (Potsdam) Wolfram Bauer (Hannover) Ingo Witt (Göttingen)

Campus Golm. Picture by F. Zanello

MICROLOCAL AND GLOBAL ANALYSIS, INTERACTIONS WITH GEOMETRY

SPEAKERS

Scientific Program					
Times	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Chair	*Paycha*	*Paycha*	*Witt*	*Baudoin*	*Schrohe*
9:00–9:50	**Bär**	**Witt**	**Lee (online)**	**Popivanov**	**Jäh**
10:00–10:50	**Fedosova**	**Namboodiri**	**Federico**	**Benoit**	**Dasgupta (online)**
10:50–11:20	**Break**				
11:20–12:10	**Mohsen (online)**	**Baudoin**	**Contini**	**Schrohe**	**Fulsche**
12:10–13:30	**Lunch break**				
Chair	*Bauer*	*Zanello*	*Fedosova*	*Federico*	
13:30–14:20	**Piazza**	**Gómez Cobos**	**Han**	**Klein**	
14:30–15:20	**Fürst**	**Toft (online)**	**Shen**	**Wei**	
15:20–15:50	**Break**				
15:50–16:40	**Savale**	**Tarama**	**Rodriguez**		
16:50–17:40					
18:00		**Conference dinner**			

- **Venue**: Building 10, Campus Golm
- **Conference dinner**: Augustiner im Bürgerbahnhof
  Sanssouci Park, Geschwister-Scholl-Straße 37, Potsdam
- **Zoom login data**: https://uni-potsdam.zoom.us/j/8733076196?omn=69157865736
  Meeting ID: 873 307 6196 Passcode: 29634559

Abstracts

Christian Bär

Universität Potsdam

TITLE: **A holographic index theorem and applications to scalar curvature geometry.**

In the first part of the talk I will discuss a "holographic" index theorem for compact manifolds with boundary. It relates the index of a boundary value problem to the index of an operator on the boundary. It will then be applied to scalar curvature geometry. We will show a rigidity theorem for scalar curvature on certain warped product spaces. This implies, in particular, Llarull's theorem for the punctured sphere. This is based on joint work with Simon Brendle, Bernhard Hanke, and Yipeng Wang.

Fabrice Baudoin

Aarhus University

TITLE: **The Parabolic Anderson on fractals and its asymptotic properties.**

A metric measure space equipped with a Dirichlet form is called recurrent if its Hausdorff dimension is less than its walk dimension. In bounded domains of such spaces we study the parabolic Anderson models

$$\partial_t u(t,x) = \Delta u(t,x) + \beta u(t,x)\, \dot{W}_\alpha(t,x)$$

where the noise W_α is white in time and colored in space when $\alpha > 0$ while for $\alpha = 0$ it is also white in space. Both Dirichlet and Neumann boundary conditions are considered. Besides proving existence and uniqueness in the Itô sense we also get precise L^p estimates for the moments and prove intermittency properties of the solution as a consequence. Our results for instance apply in fractals like the Sierpiński gasket for which we study the scaling invariance properties of our models.

Antoine Benoit

Université du Littoral Côte d'Opale

TITLE: **Hyperbolic boundary value problems in corner domains: well-posedness and geometric optics expansions.**

In this talk, I will consider the well-posedness issue for hyperbolic boundary value problems in corner domains. Such problems have a rather long story since they appear back the 70's in the works of Osher and Sarason-Smoller. Meanwhile they were somewhat forgotten until the recent contributions of Huang-Temam, Halpern-Rauch and the author. For such problems, imposing the uniform Kreiss-Lopatinskii condition (that is to say the condition ensuring the well-posedness for the half-space problem) on each side of the corner is not sufficient for strong well-posedness, which comes as no surprise.

In the talk, I will first describe the new condition for well-posedness of Osher. This condition, which involves Fourier integral operators, is, unfortunately very difficult to handle. Consequently, at present time it only gives rise to a well-posed solution with a (non explicit !) number of losses of derivative. We are therefore still far from a strong well-posednesss result. Using geometric optics expansions, I will further show that a microlocalized version of Osher's condition appears when constructing the expansion. This yields a strong hint towards the fact that Osher's condition is effectively the good condition to consider. The construction of rigorous geometric optics expansions for corner problems is a rather delicate problem and in this talk, I will describe the main issue, namely self-interaction of the phases involved by the corner geometry.

Alessandro Contini

Leibniz Universität Hannover

TITLE: **Isomorphisms of the algebra of SG-pseudo-differential operators.**

We generalise a result of Duistermaat and Singer about order-preserving isomorphisms of pseodu-differential operators to the case of the *SG*-algebra. In particular, we show that they are given by conjugation with Fourier integral operators with inhomogeneous phase functions of a very specifc form, coming from a singular symplectic structure on the compactified cotangent bundle.

Aparajita Dasgupta

Indian Institute of Technology, Delhi, India

TITLE: **Non-Harmonic Analysis of Weighted Pseudo-differential Operators.**

In the framework of non-harmonic analysis of boundary value problems on a manifold with boundary as introduced by Ruzhansky and Tokmagambetov Int. Math. Res. Not. IMRN, (12), 3548-3615, 2016) I will present the global symbolic calculus of weighted pseudo-differential operators generated by a boundary value problem for a given (not necessarily self-adjoint or elliptic) differential operator. Here elements of a non-self-adjoint distribution theory and the corresponding biorthogonal Fourier analysis will be shown without any assumptions on the regularity of the boundary which is allowed to have arbitrary singularities. I will provide symbolic calculus, describe the notion of ellipticity, and show a characterization of compact operators and Riesz operators associated with the weighted symbol class. As an application, a unique strong solution of the corresponding parabolic pseudo-differential equation will be presented.

(This is joint work with V. Kumar, L. Mohan, and S.S. Mondal)

Serena Federico

Universitá di Bologna

TITLE: **Weyl Calculus on graded groups.**

The goal of this talk is to present a Weyl pseudo-differential calculus on general graded nilpotent Lie groups established recently in collaboration with D. Rottensteiner and M. Ruzhansky. I will fist recall the basics of the Euclidean Weyl pseudo-differential calculus in order to set the stage for the Lie group setting. Next, I will describe a class of symmetric pseudo-differential calculi on general graded nilpotent Lie groups, and finally I will show which one among all the presented quantizations is the candidate Weyl quantization according to the properties of the corresponding calculus. At the end of the talk we will see that the identified candidate quantization is effectively the Weyl quantization in the case of the Heisenberg group.

Ksenia Fedosova

Universität Freiburg

TITLE: **Selberg zeta function twisted with non-unitary twists.**

The Selberg zeta-function is an analogue of the usual zeta function, which uses the lengths of simple closed geodesics on a surface instead of the prime numbers. In this talk, we consider generalizations of the modular surface and their flat vector bundles. If a flat vector bundle is associated to a unitary representation, then its Selberg zeta function is well-defined and admits a meromorphic continuation to the whole complex plane with zeroes connected to the spectral information of the Laplace operator on the flat vector bundle. If the representation is not unitary, but has a property of not growing/decaying too fast in cusps, the Selberg zeta function can still be defined – however, its domain of the convergence becomes smaller.

If we consider even larger class of the representations, the Selberg zeta function fails to converge at all. In this talk we would like to investigate if it is possible to define the Selberg zeta function in this situation with the help of methods from the thermodynamical formalism and how the problem is connected to the fact that polylogarithms are, in general, not single-valued functions.

Robert Fulsche

Leibniz Universität Hannover

TITLE: **Spectral estimates for Schrödinger operators with singular potentials on the real line.**

We present results on the spectrum of operators of the form $-\frac{d^2}{dx^2} + \mu$ on $L^2(\mathbb{R})$, where μ is a suitable measure. More precisely, we will discuss a two-sided estimate for the eigenvalue counting function of this operator, in terms of Otelbaev's function (which is, in some sense, a regularization of the potential). As applications, we present a version of Molchanov's criterion for discrete spectrum, as well as criteria for Schatten class membership of the resolvents. This is based on a paper which was joint work with Medet Nursultanov. If time permits, we might also present some results from an ongoing related project which is jointly with Medet and Grigori Rozenblum.

Oliver Fürst

Leibniz Universität Hannover

TITLE: **Trace and Index of Dirac-Schrödinger Operators on Open Space with Operator Potentials.**

We develop trace and index formulas for a Dirac-Schrödinger operator on open space of dimension d=3 with a potential given by a family of self-adjoint unbounded operators acting on an infinite dimensional Hilbert space

H. The index formula generalizes the index theorem by C.Callias to unbounded operators, while the trace formula generalizes existing trace formulas of d=1 to higher dimensions.

Santiago Gómez Cobos

Universiteit Gent

TITLE: **L^p-bounds in Safarov pseudo-differential calculus on compact manifolds.**

Pseudo-differential operators on smooth manifolds are defined classically by means of local coordinates using the well-known theory for $\mathbb{R}^n$. Since we would like to have invariance under change of coordinates this will naturally imply some restrictions on the classes that we can construct, particularly when we are faced with the restriction $\rho > 1/2$ on the classes $S^m_{\rho,\delta}$. Safarov introduced a pseudo-differential calculus on smooth manifolds using a linear connection ∇ in order to obtain a global symbol. Using this construction and assuming that the connection is symmetric he was able to weaken the aforementioned restriction to $\rho > 1/3$. We will introduce the Safarov pseudo-differential calculus and we will present a result on $L^p - L^p$ estimates for this calculus.

Zhicheng Han

Georg-August-Universität Göttingen

TITLE: **Spectra of Lie groups and application to L^2-invariants.**

In this talk, I will explore the Laplace operator and Dirac operator on semisimple Lie groups. While the parallel problem on symmetric spaces has been well-studied in the last century, the corresponding problem is much less understood in general homogeneous spaces. We will examine the obstacles in extending existing techniques and discuss how some of them can be resolved in the case of group manifolds. Towards the end, we will see how the spectra data shall aid in computing certain topological L^2-invariants.

Christian Jäh

Georg-August-Universität Göttingen

TITLE: **Maximal Sobolev Regularity at Radial Points.**

In this talk, we present ongoing work concerning maximal Sobolev regularity at radial points. This work continues the classical study of Guillemin and Schaeffer. We will shall discuss the progress it provided in handling normal forms. The normal form of the operator is of second order and the main task is to construct an appropriate parametrix. We will discuss precise regularity results for solutions to pseudodifferential equations with real principal symbol near isolated radial points, microlocally in the scale of Sobolev spaces H^s. Compared to operators of real principal type, the new phenomenon of *maximal Sobolev regularity* appears and the index of maximal Sobolev regularity can be effectively computed in terms of the sub-principal symbol of the operator under consideration.

This is joint work with Ingo Witt and Michael Ruzhansky.

Christian Klein

Université de Bourgogne

TITLE: **Direct and inverse scattering for Davey-Stewartson equations.**

The integrable Davey-Stewartson (DS) equations are nonlinear Schrödinger equations in 2 spatial dimensions. Solutions to the DS equations can be constructed by solving a Dirac system which also is used in the context of electric impedance tomography. We describe an approach to the Dirac system for large values of the spectral parameter for potentials in suitable Besov spaces. A polarization approach is applied. This is work with J. Sjöostrand and M. Zerzeri.

Gihyun Lee (online)

Universiteit Gent

TITLE: **Pseudodifferential operators on noncommutative tori and their mapping properties.**

Noncommutative tori are the most intensively studied noncommutative spaces in Alain Connes' noncommutative geometry program and arise in various parts of mathematics and mathematical physics. Pseudodifferential calculus on noncommutative tori was introduced in early 1980s by Connes, and it has emerged as an indispensable tool in the recent study of differential geometry of noncommutative tori.

There are various ways of constructing pseudodifferential calculus on noncommutative tori. In this talk, I will provide an overview of these various approaches, discuss some basic properties of these pseudodifferential operators and explain how they are utilized in the study of differential geometry of noncommutative tori. Furthermore, I will also explain the boundedness of pseudodifferential operators on noncommutative L_p-spaces associated with noncommutative tori.

This talk is based on joint works with Hyunsu Ha, Raphaël Ponge and Vishvesh Kumar.

Omar Mohsen (online)

Université Paris-Saclay

TITLE: **Tangent groupoid approach to pseudodifferential calculus.**

A few years ago, Debord and Skandalis, and Yuncken and van-Erp gave a way to study classical pseudodifferential operators using Connes' tangent groupoid. This approach has various advantages over the classical symbolic approach due to Hormander. In my talk, I will recall the notion of a groupoid and focus on the tangent groupoid as the main example. I will then present the approach of Yuncken and van-Erp, and how it is related to the classical approach of Hörmander.

The talk will end with a presentation of our recent work, which shows that the Wodzicki residue and the Kontsevich-Vishik trace fit very nicely with this new approach to pseudodifferential calculus.

Narayanan Namboodiri

Cochin University of Science and Technology

TITLE: **Szego-Type Limit Theorems for self-adjoint operators-Recent developments.**

Arveson addressed the problem of numerical analysis of the essential spectrum of a bounded self-adjoint operator on a separable, complex Hilbert space, using the eigenvalues of its finite-dimensional compressions in a suitable orthonormal basis. He proved many interesting results, such as Szego-type limit theorems for self-adjoint bounded operators on separable Hilbert Spaces with applications to discretized, one-dimensional Schrödinger operators. More generally, a unified approach to this problem using the concepts of Folner nets/ sequences and Folner C^*-algebras have been initiated by various researchers. This lecture is a short survey of the recent developments in this area of research.

Paolo Piazza

Sapienza Università di Roma

TITLE: **The signature operator on a Witt space: analytic Gysin maps in K-homology.**

After a brief introduction to the signature operator on a Witt space and its K-homology class (also known as the analytic orientation class), I will present recent work with Pierre Albin and Markus Banagl in that context. I will first define an analytic Gysin map in K-homology for a fiber bundle map of Witt spaces and for a normally non-singular inclusion of Witt spaces . I will then state our main result which states that these Gysin maps preserve the orientation class. Our work employs KK-theory and the edge pseudodifferential calculus in an essential way.

Nedyu Popivanov

Bulgarian Academy of Sciences

TITLE: **Protter-Morawetz multidimensional BVP problems, exponential type singularity of the generalized solutions.**

About sixty years ago Murray Protter presented a multidimensional variant of the famous Guderley-Morawetz plane problem for hyperbolic-elliptic equations that had been studied by Morawetz, Lax and Phillips. This problem now is known as Protter - Morawetz problem. Protter also gives an analogous statement of multidimensional versions of such kind of problems for the wave equation. Even in this case it appeared later that the analogous boundary value problem is strongly overdetermined in a classical statement and is not of Fredholm type. We will present now some results in the frame of generalized solution statements with possible high order of singularity right-hand sides of the wave equation. This singularity is fixed on the vertex of the light cone and dos not going away of it. In the (3+1)-D case we show some possible extensions of the generalized solution outside of the wave cone which could explain such kind of singularity. Some open questions arose, for example: what kind of extensions appears in the case of exponential growth of singularity, when is not possible to work in the distribution frame? The talk is based on the joint work with Ingo Witt, University of Göttingen and Todor Popov, University of Sofia.

Miguel Rodriguez

Leibniz Universität Hannover

TITLE: **Commutative Toeplitz Banach Algebras and Gelfand Theory.**

In this talk, we provide an overview of the theory of commutative Banach algebras generated by Toeplitz operators on the Bergman space over the unit ball and present some recent results. We will discuss the construction of these algebras, presenting non-trivial examples and explaining how the geometry of the domain relates to the properties of the corresponding algebras. Furthermore, we will explore the application of Gelfand Theory in obtaining structural information from these algebras, specifically characterizing their semi-simplicity and spectral invariance.

Nikhil Savale

Universität zu Köln

TITLE: **Harmonic spinors in odd dimension.**

We prove that any odd dimensional spin manifold admits a metric with a non-trivial harmonic spinor. This completes the odd dimensional case of the problem following the work of Hitchin and Bär

Wei Chuan Shen

Universität zu Köln

TITLE: **Semi-classical spectral asymptotics of Toeplitz operators on CR manifolds.**

Let X be a compact strictly pseudoconvex embeddable CR manifold and let T_P be the Toeplitz operator on X associated with a first order pseudodifferential operator P. We consider the operator $\chi_k(T_P)$ defined by functional calculus of T_P, where χ is a smooth function with compact support in the positive real line and $\chi_k(\lambda) := \chi(k^{-1}\lambda)$. We will show that the kernel of $\chi_k(T_P)$ is a semi-classical Fourier integral modulo a k-negligible smooth kernel. Time permitting, I will give some applications that provide CR analogues of asymptotics for powers of line bundles in complex geometry.

Elmar Schrohe

Leibniz Universität Hannover

TITLE: **Asymptotics of the Porous Medium Equation on Manifolds with Conical Singularities.**

The Porous Medium Equation (PME) is a non-linear variant of the heat equation. The problem is to find a solution u to the equations

$$\dot{u}(t,x) - \Delta u^m(t,x) = f(t,u), \quad u(0,x) = u_0(x).$$

The name is derived from the fact that it describes - among other phenomena - the flow of a gas in a porous medium. Here, u is the density of the gas, t is a time parameter, x the space variable, m is a positive constant, and f is a forcing term; for simplicity we assume it to be Lipschitz in t and holomorphic in u. Finally u_0 the value of u.

We consider the PME on a compact manifold with conical singularities for a strictly positive initial value u_0. In the talk I will explain how maximal regularity techniques can be used to establish the existence of a solution in suitable cone Sobolev spaces with asymptotics, what we can say about the regularity of the solution and how the geometry of the manifold near the conical points is reflected in the structure of the solution.

(Joint work with N. Roidos, Patras/GR)

Alexander Strohmaier

Leibniz Universität Hannover

TITLE: **The timelike tube theorem in quantum field theory on curved spacetimes and unique continuation.**

I will explain the timelike tube theorem, a known theorem for Wightman fields in Minkowski space, and how it can be generalised to quantum field theory in curved spacetimes. The talk will contain an introduction to quantum fields on curved spacetimes and I will link analytic microlocal spectrum conditions to the validity of the timelike tube theorem. (based on joint work with E. Witten).

Daisuke Tarama

Ritsumeikan University

TITLE: **Geodesic flows on Lie groups associated to statistical transformation models.**

A statistical transformation model is a transformation model for which a Lie group smoothly acts on the sample manifold equipped with a family of relatively invariant probability density functions. In the framework of information geometry, the Fisher-Rao (semi-definite) metric and the Amari-Chentsov cubic tensor are defined over the parameter Lie group with which the α-connections are associated. In the talk, the geodesic flows induced by these connections are studied in view of Lagrangian and Hamiltonian systems. In a special case, the relation to a certain subriemannian structure is also mentioned.

Joachim Toft (online)

Linnæus University

TITLE: **(online) Orlicz spaces and Orlicz modulation spaces: Pseudo-differential calculi and entropy estimates.**

A convex function Φ from $[0,\infty]$ to $[0,\infty]$ with properties

$$\Phi(0)=0, \qquad \lim_{t\to\infty}\Phi(t)=\Phi(\infty)=\infty,$$

is called a *Young function*. For any Young function Φ, the Orlicz space L^Φ is a Banach space, and consists of all measurable functions f such that $\Phi(t\cdot|f|)\in L^1$ for some $t>0$. The family of Orlicz spaces contain any Lebesgue space, which one recovers by choosing the function Φ adequately.

The Orlicz modulation space M^Φ is obtained by imposing L^Φ norm conditions of the short-time Fourier transforms of the involved functions and distributions. In the same way we may discuss Orlicz modulation spaces $M^{\Phi,\Psi}$ of mixed normed types. Again, by choosing the Young functions Φ and Ψ in suitable ways, $M^{\Phi,\Psi}$ becomes the classical Feichtinger's modulation space $M^{p,q}$.

In the talk we explain some basic properties and give some examples on interesting Orlicz spaces and Orlicz modulation spaces. We also explain some classical results on pseudo-differential operators acting on Lebesgue or modulation spaces. We hope that this can shed some light on how to find suitable Banach spaces when dealing with non-linear functionals.

The talk is based on joint works with A. Gumber, E. Nabizadeh Morsalfard, N. Rana, S. Öztop and R. Üster.

Yawei Wei

Nankai University, China

TITLE: **Mean field games systems with degenerate diffusions.**

In this talk, we will first introduce the mean field games and the partial differential systems of mean field games, including the motivation and the research history. Then we will discuss the degenerate nonlinear PDE systems arising from the mean field games, where the generic player may have a forbidden direction at some point. Here we prove the existence and uniqueness for the PDE systems, which describe the Nash equilibria in the games.

Ingo Witt

Georg-August Universität Göttingen

TITLE: **On elliptic problems with changing boundary conditions.**

We study boundary problems for elliptic differential and pseudodifferential operators, where the boundary is divided into two parts and different elliptic boundary conditions are imposed along the two parts of the boundary. An archetypal example is the Zaremba problem for the Laplace operator in a bounded domain in $\mathbb{R}^n$ with the Dirichlet and the Neumann boundary condition imposed in different parts of the boundary. We approach this problem following a strategy proposed by B.-W. Schulze using a boundary reduction. This boundary reduction leads to an edge-degenerate boundary problem in the boundary which in turn is discussed in the light of a calculus for edge-degenerate boundary problems recently proposed by X.-C. Liu, Z.-P. Ruan, and myself. This is joint work with

Zhuoping Ruan (Nanjing University).