Microlocal and Global Analysis, Interactions with Geometry

20-24 February 2023, Potsdam

Organizers:

Sylvie Paycha (Potsdam) Chand Devchand (Potsdam) with the valuable help of Sylke Pfeiffer

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MICROLOCAL AND GLOBAL ANALYSIS, INTERACTIONS WITH GEOMETRY

SPEAKERS

Scientific Program					
Times	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Chair					
9:00–9:50	**no talk**	**Bei**	**Popivanov**	**Fedeosova**	**Pankrashkin**
9:50–10:30	Break				
10:30–11:20	**Wahlberg**	**Anop**	**Ruan**	**Tarama**	**Mattar Marriaga**
11:30–12:20	**Martini**	**Correa**	**Cardona**	**Vega-Molino**	**Contini**
12:20–13:20			Lunch Break		**12:20 - 13:10 Schrohe**
13:20–13:50		**Mazlum (book review)**			
Chair					
13:50–14:40	**Fulsche**	**Klein**	**Longhi**	**Piazza**	
14:50–15:40	**Berge**	**Rowlett**	**Gihyun Lee**	**Richtsfeld**	
15:40–16:20	Break				
16:20–17:10	**Waters**	**Savale**	**Kutsenko**	**Schmitt**	

Abstracts

Anna Anop

Math. Inst. Goettingen, Inst. of Math. NAS of Ukraine

TITLE: **On elliptic problems in Sobolev spaces of generalized smoothness.**

The talk is devoted to a new theory of elliptic problems in an extended Sobolev scale. This scale consists of some Hilbert distribution spaces whose generalized smoothness is given by a positive function that depends radially on the frequency variables and varies O-regularly at infinity. These spaces are obtained by the quadratic interpolation between Sobolev spaces and allow a definition on compact smooth manifolds by localization. We discuss theorems on solvability of elliptic boundary-value problems and regularity of their solutions in these spaces. The case of rough boundary data is included. We give an application to some elliptic problems with white noise on the boundary. These results were obtained together with Prof. Robert Denk and Prof. Aleksandr Murach. Anop A., Denk R., Murach A. Elliptic problems with rough boundary data in generalized Sobolev spaces. Comm. Pure Appl. Anal. 20 (2021), no. 2, 697-735.

Francesco Bei

Sapienza, Università di Roma

TITLE: **On weakly Kähler hyperbolic manifolds and geometric applications.**

Kähler hyperbolic manifolds were introduced by Gromov around thirty years ago. In his seminal paper he showed that this class of Kähler manifolds enjoys several remarkable properties: for instance they are of general type, Kobayashi hyperbolic and their L^2-Hodge numbers $h_(2)^{p,q}$ are positive if and only if $p+q = dim(M)$. In this talk I will report about a recent joint work with S. Diverio, P. Eyssidieux and S. Trapani where we introduced some weak versions of Kähler hyperbolicity. I will show how some of the properties proved by Gromov for Kähler hyperbolic manifolds remain true in our more general setting and I will explain various geometric applications. In particular I will describe how these ideas allow to verify some aspects of the Lang conjecture for Kähler hyperbolic manifolds.

Stine Marie Berge

Leibniz Universität Hannover

TITLE: **Affine Quantum Harmonic Analysis.**

In time-frequency analysis the time-frequency representation on the Heisenberg group gives rise to the short-time Fourier transform (STFT). The Wigner distribution in quantum mechanics can be written as the Fourier transform of the STFT. Using the Wigner distribution we can define the Weyl quantization, giving us a way to associate

operators with functions. It turns out that this quantization scheme works very well in tandem with operator versions of convolutions. Using this framework, it is possible to express many objects in time-frequency analysis, e.g., localization operators. In much the same way, we show that one can use the wavelet representation in wavelet analysis to obtain an affine Wigner distribution and an affine quantization scheme on the affine group. What we will see is that many of the properties present in time-frequency analysis have analogues in wavelet theory, e.g. Moyal identities. However, due to the non-unimodularity of the affine group, both the left and the right Haar measures have an important role in the theory.

Duvan Cardona Sanchez

Ghent University

TITLE: **Estimates for sums of eigenfunctions on compact manifolds.**

In this talk we present some results about the growth of sums of eigenfunctions on compact manifolds. In the case of general compact manifolds criteria are presented in terms of the positivity of the principal symbol. As for compact Lie groups, criteria are also given in terms of the positivity of the matrix-valued symbol. Applications to control theory are presented.

Alessandro Contini

Leibniz Universität Hannover

TITLE: **Canonical transformations on the scattering manifolds and SG-calculus.**

The relation between Fourier Integral Operators in SG classes and the symplectic structure of a scattering manifold has yet to be explored fully. While some work on singular Lagrangian distributions on a single manifold has appeared, the case of the graph of a symplectic map still remains unsolved. In this talk I will introduce a notion of singular symplectomorphism coming from the problem of determining the nature of order-preserving isomorphisms of the SG-algebra, and show how this can be regarded as a singular Lagrangian submanifold by finding local parametrisations in terms of phases chosen in SG classes.

Santiago Correa

Univerität Göttingen

TITLE: **Energy estimates for boundary value problems of WR type.**

Given a constantly hyperbolic differential operator L in a domain with non-characteristic boundary for L, we can classify the boundary conditions into three generic classes according to the Lopatinskii condition (LC), namely,

(1) when (LC) holds,
(2) when (LC) fails,
(3) when (LC) holds but not uniformly in a way to be explained in the talk.

There is an ample discussion of (1) in the literature and the results are well-studied. Case (2) can be proven to be unstable, and therefore there is no hope for any satisfactory theory. In this talk, we shall deal with the last situation and derive energy estimates on the solutions, keeping in mind potential applications to nonlinear problems.

Ksenia Fedeosova

Universität Freiburg

TITLE: **Whittaker Fourier type solutions to differential equations arising from string theory.**

In this talk, we consider functions appearing in the graviton scattering in the string theory. These functions, sometimes referred to as generalized Eisenstein series, are defined on the modular surface (that is, the quotient of the hyperbolic upper half-plane by the action of the modular group) and satisfy the inhomogeneous Laplace equation. We study the Fourier expansions of such functions at the cusp. Interestingly enough, the boundary conditions of such functions provide new number-theoretical formulas involving divisor functions. This is a joint work with Kim Klinger-Logan; a part of it can be found here: https://arxiv.org/abs/2209.09319

Robert Fulsche

Leibniz Universität Hannover

TITLE: **News from Quantum Harmonic Analysis.**

Quantum Harmonic Analysis in the sense of R. Werner's paper "Quantum Harmonic Analysis on phase space" from '84 can be seen as a framework for extending notions from classical harmonic analysis for functions, such as convolutions and Fourier transforms, to operations involving both functions and operators. Having been introduced for considerations in quantum mechanics, the theory has been ignored by mathematicians for a long time. In recent years, these ideas were taken up by people working in both operator theory and time-frequency analysis, proving that it is a now field of significant mathematical interest as well. In this talk, we will give a brief introduction to the notions and methods of Quantum Harmonic Analysis and will sketch its connection to the theory of Toeplitz operators and to time-frequency analysis. If time permits, we will shortly discuss one of the speaker's recent projects related to Quantum Harmonic Analysis.

Christian Klein

Université de Bourgogne,Dijon

TITLE: **Complex geometrics optics solutions to d-bar equations.**

Complex geometric optics solutions to a system of d-bar equations play an important role in the context of electrical impedance tomography (EIT) and the scattering theory of the integrable Davey-Stewartson II equations. Whereas the situation is well understood for Schwartz class potentials, the case of potentials with compact support that is important in EIT and for the study of so-called dispersive shock waves is challenging. A detailed discussion of the

case of potentials q with compact support on some domain Ω with smooth strictly convex boundary is presented for large values of the spectral parameter k. Improved results are given for the reflection coefficient, i.e., the scattering data.

Anton Kutsenko

Katholische Universität Eichstätt-Ingolstadt

TITLE: **How random permutations create waves.**

We consider a simple donation pyramid model, the chain of citizens, where every second citizen gives her/his coin to a randomly chosen left or right neighbour, and, after that, leaves the chain. The remaining citizens take the places of the leaving citizens, ordered or with random permutations (disordered). The process is repeated many times. The main result is that in the ordered case the distribution of coins has a linear growth, while in the disordered case it is also linear but perturbed by some oscillations having a complex fractal structure. Generally, such types of systems can model the following phenomena: movement of lithospheric plates; ice jam formation; donation pyramid; merging of cells in biological organisms; etc.

Gihyun Lee

Ghent University

TITLE: **The semiclassical limit for quantum systems and pseudodifferential calculus.**

Approximating quantum mechanical equations by classical equations for macroscopic systems has been a topic of great interest since the dawn of quantum mechanics. In this talk, I will discuss how to approximate the quantum mechanical equation which describes the time evolution of physical states (a.k.a. the Lindblad master equation) by a classical equation. The Lindblad master equation is formulated in terms of a super operator, i.e., a linear map between spaces of linear operators, so a novel pseudodifferential calculus of super operators will be introduced to lay down a rigorous functional analytic ground for understanding the approximation described above. Based on joint work with M. Lein.

Rubens Longhi

Universität Potsdam

TITLE: **Wavefront sets of different regularity via the Radon transform.**

We develop a new formulation of the wavefront set of a distribution which generalizes the standard results using the Radon transform. The usual notion of smooth wavefront set captures the cotangent directions along which a distribution on a manifold locally fails to be smooth. Smoothness near such a direction u can be rephrased by requiring for any test function φ with compact support, the Radon transform $R_\omega(\varphi u)$ to be smooth for ω in the cotangent unit sphere, this leading to an alternative definition of the wavefront set by means of the Radon transform.

We hence define a new notion of wavefront set requiring the localized Radon transform $R_\omega(\varphi u)$ to lie in a space of functions $\mathscr{F}(\mathbb{R})$ to be specified. New spaces of local regularity on manifolds arise from this construction, which reduces the analysis to that of spaces of functions on $\mathbb{R}$. In particular, we establish conditions on families of spaces $\mathscr{F}(\mathbb{R})$ on the real line such that microlocal regularity results hold true.

Alessio Martini

Politecnico di Torino

TITLE: **A Fourier integral operator approach to the sub-Riemannian wave equation.**

Let L be a sub-Laplacian on a sub-Riemannian manifold of dimension n. We show that the ranges of validity of spectral multiplier estimates of Mihlin-Hörmander type and wave propagator estimates of Miyachi-Peral type for L cannot be wider than the corresponding ranges for the Laplace operator on R^n - despite the lack of ellipticity of L. The proof hinges on a Fourier integral representation for the wave propagator associated with L and nondegeneracy properties of the sub-Riemannian geodesic flow.

Gisel Mattar Marriaga

Universität Göttingen

TITLE: **On the principal symbol of Fourier integral operators with complex phase.**

Fourier Integral Operators with complex phase function arise naturally in applications, especially when dealing with parametrices for operators that have non-real principal symbols. It is then useful to have a theory for the study Fourier Integral Operators with complex phase. We followed the approach proposed by Melin and Sjöstrand (1975). Among other things, the authors studied the composition under the assumption of transverse intersection, and gave an incomplete description of the principal symbol of the resulting distribution. We now extend their work, by presenting an alternative construction of the principal symbol. With our characterization of the principal symbol, we are able to extend their result by providing an explicit formula for the principal symbol after composition.

Konstantin Pankrashkin

Universität Oldenburg

TITLE: **Curvature contribution to the essential spectrum of Dirac operators with critical shell interactions.**

We discuss the spectral properties of three-dimensional Dirac operators with critical combinations of electrostatic and Lorentz scalar shell interactions supported by a compact smooth surface. It turns out that the criticality of the interaction may result in a new interval of essential spectrum. The position and the length of the interval are explicitly controlled by the coupling constants and the principal curvatures of the surface. This effect is completely

new compared to critical situations for lower dimensions or special geometries considered up to now, in which only a single new point in the essential spectrum was observed. Based on joint work with Badreddine Benhellal (Oldenburg).

Paolo Piazza

Sapienza Università di Roma

TITLE: **Positive scalar curvature metrics on spin pseudomanifolds.**

In the case of compact manifolds without boundary there are strong analogies between the signature operator on an orientable manifold and the spin-Dirac operator on a spin manifold. A pseudomanifold, which generalises the notion of manifold, isa precise realisation of the general idea of a manifold with singularities and Witt pseudomanifolds are pseudomanifolds with specific homological properties. The signature operator on a Witt pseudomanifold has been proved to enjoy very strong properties, to the point that once a certain dictionary is applied (oriented bordism into Witt bordism, de Rham cohomology into L^2-cohomology, singular homology into intersection homology) many of the familiar properties on compact manifolds can be stated verbatim for the signature operator on Witt pseudomanifold. In this talk I will explain how and why this principle breaks down for the spin-Dirac operator on a spin pseudomanifold. In particular, I will explain why the situation for this operator is much more delicate than for the signature operator and how this has an impact on the problem of existence of conic metrics of positive scalar curvature. This is joint work with Boris Botvinnik and Jonathan Rosenberg and also, more recently, with Pierre Albin and Jesse Gell-Redman.

Nedyu Popivanov

Bulgarian Academy of Sciences

TITLE: **Protter-Morawetz multidimensional BVP problems, exponential type singularity of the generalized solutions.**

About sixty years ago Murray Protter presented a multidimensional variant of the famous Guderley-Morawetz plane problem for hyperbolic-elliptic equations that had been studied by Morawetz, Lax and Phillips. This problem now is known as Protter - Morawetz problem. Protter also gives an analogous statement of multidimensional versions of such kind of problems for the wave equation. Even in this case it appeared later that the analogous boundary value problem is strongly overdetermined in a classical statement and is not of Fredholm type. We will present now some results in the frame of generalized solution statements with possible high order of singularity right-hand sides of the wave equation. This singularity is fixed on the vertex of the light cone and dos not going away of it. In the (3+1)-D case we show some possible extensions of the generalized solution outside of the wave cone which could explain such kind of singularity. Some open questions arose, for example: what kind of extensions appears in the case of exponential growth of singularity, when is not possible to work in the distribution frame? The talk is based on the joint work with Ingo Witt, University of Göttingen and Todor Popov, University of Sofia.

Alberto Richtsfeld

Universität Potsdam

TITLE: **A local index density for the Rarita-Schwinger operator.**

In this talk, I will present a modified version of the proof of Atiyah, Patodi and Bott for the local index theorem, which generalizes to a wider class of geometric differential operators. This will in particular enable us to identify the local index density of the Rarita–Schwinger operator.

Julie Rowlett

Chalmers Tekniska Högskola

TITLE: **The diversity theorems in theory and practice.**

Diversity can be beneficial in many contexts. In business, research, and education diversity can not only increase creativity by providing a wider variety of perspectives but also increase impact through a larger network. In biology, biodiversity is often correlated to the overall health of an ecosystem, perhaps because a diverse ecosystem has a greater capacity for adaptive responses to new challenges. In finance, a cornerstone of modern portfolio theory is diversification of investments. I will first discuss joint work with CJ Karlsson and M Nursultanov. Our main results are 'the diversity theorems' which show that teams of diverse individuals are strong in competition with other teams of individuals. The proof of these theorems are based on game theoretic models that interpolate between individual-level interactions and collective-level ramifications. In joint work with J Guldholm, J. Klünder, and J Stålberg we tested the theoretical predictions using a tremendous amount of data from professional football (soccer). In conclusion, I will discuss the results of our joint work and further investigations.

Zhuoping Ruan

Nanjing University

TITLE: **: Semilinear elliptic equations with a uniform boundary blowup.**

We prove the existence of solutions for a class of nonlinear equations on a compact manifold with boundary defined in terms of an elliptic edge-degenerate differential operator. Our result generalizes previous results of several authors, e.g., a result of Alan Lazer and Patrick McKenna (1994) for the Laplacian in a bounded domain in R^N. While we use microlocal methods, in particular, a calculus for boundary problems for edge-degenerate operators (our joint work with Xiaochun Liu (Wuhan)), those authors used comparison principles and were bound to the case $m = 2$. This is joint work with Ingo Witt (Göttingen).

Nikhil Savale

University of Collogne

TITLE: **Quantitative version of Weyl's law.**

We prove a general estimate for the Weyl remainder of an elliptic, semiclassical pseudodifferential operator in terms of volumes of the recurrence sets for its Hamilton flow; thereby quantifying earlier results of Volovoy. Our result particularly gives quantitative estimates for Berard's Weyl remainder in terms of the maximal expansion rate and topological entropy of the geodesic flow.

Philipp Schmitt

Leibniz Universität Hannover

TITLE: **The index of hypoelliptic Shubin type operators.**

In recent years, the index problem for certain classes of hypoelliptic operators received considerable attention. These operators become elliptic in adapted symbol calculi, like the Heisenberg calculus and its generalizations to other filtered manifolds, where the notion of order is altered. On compact manifolds, the index of such operators was computed by van Erp and many other authors. In this talk, we compute the index of hypoelliptic Shubin type operators on graded Lie groups by taking a "classical limit", generalizing work by Elliott–Natsume–Nest for elliptic Shubin type operators on $\mathbb{R}^n$. A key step is the construction of a higher cyclic cocycle, needed to compute the classical limit of a certain trace. This is work in progress, joint with Eske Ewert and Ryszard Nest.

Elmar Schrohe

Leibniz Universität Hannover

TITLE: **Order Preserving Automorphisms of Shubin Type Pseudodifferential Operators .**

A Shubin symbol of order $m \in \mathbb{Z}$ is a smooth function $a = a(x,\xi)$ on $\mathbb{R}^n \times \mathbb{R}^n$ that satisfies the estimates

$$|\partial_\xi^\alpha \partial_x^\beta a(x,\xi)| \le C_{\alpha,\beta}(1+|x|^2+|\xi|^2)^{(m-|\alpha|-|\beta|)/2}$$

for all multi-indices α, β with suitable constants $C_{\alpha,\beta}$. We additional assume the symbol a to be classical, i.e., it has an asymptotic expansion

$$a(x,\xi) \sim \sum_{j=0}^{\infty} a_{m-j}(x,\xi)$$

with a_{m-j} homogeneous of degree $m-j$ in (x,ξ) for $|(x,\xi)| \ge 1$. We denote by Ψ^m the space of all pseudodifferential operators with Shubin type symbols of order m and by Ψ the corresponding union over all m.

Following up on a remarkable result of Duistermaat and Singer (Comm. Pure Appl. Math. XXVIV:39-47, 1976) for the case of the usual pseudodifferential operators we determine the order preserving automorphisms of Ψ. We obtain the following:

Theorem. *Let $J : \Psi \to \Psi$ be an algebra isomorphism that satisfies $J(\Psi^m) = \Psi^m$ for every $m \in \mathbb{Z}$. Then*

$$J(A) = F^{-1}AF, \quad A \in \Psi,$$

for a Shubin type Fourier integral operator F which is unique up to multiplication by a nonzero constant.

At first glance, this looks very much like the statement of Duistermaat and Singer; there are, however, significant analytic and geometric differences.

Joint work in progress with Robert Hesse (Hannover) and Ryszard Nest (Copenhagen).

Daisuke Tarama

Ritsumeikan University

TITLE: **Subriemannian geodesic flows of the 7-dimensional sphere.**

This talk deals with the subriemannian geodesic flows over the 7-dimensional sphere. Trivializable (bracket-generating) subriemannian structures of different ranks are given by means of Clifford representations. The geodesic flows are formulated as Hamiltonian systems on the cotangent bundle to the sphere and their complete integrability is proved on the basis of Thimm's method proposed around 1981. The talk is based on the collaborations with Wolfram Bauer and Abdellah Laaroussi.

Gianmarco Vega-Molino

Universitetet i Bergen

TITLE: **Local Heat Invariants of the sub-Laplacian on H-type Foliations.**

H-type foliations are a broad class of step 2 sub-Riemannian manifolds that carry an H-type group structure on the tangent space at each point. Equipping the manifold with the Bott connection we consider the scalar horizontal curvature as well as a new local invariant induced from the vertical distribution. We extend recent results on the small-time asymptotics of the sub-Riemannanian heat kernel on quaternion-contact (qc-)manifolds due to A. Laaroussi and we express the second heat invariant in sub-Riemannian geometry as a linear combination of the mentioned invariant. The use of an analog to normal coordinates in Riemannian geometry that is well-adapted to the geometric structure of H-type foliations allows us to consider the pull-back of Korányi balls. We explicitly obtain the first three terms in the asymptotic expansion of their Popp volume for small radii.

This talk is based on a joint work with Irina Markina (Universitetet i Bergen), Wolfram Bauer (Leibniz Universität Hannover) and Abdellah Laaroussi (Leibniz Universität Hannover).

Patrik Wahlberg

Politecnico di Torino

TITLE: **Global microlocal analysis on Gelfand-Shilov spaces.**

We define an anisotropic global wave front set for Beurling type Gelfand-Shilov ultradistributions. We show that a condition on the wave front set of the Schwartz kernel of a linear operator implies its continuity on certain Gelfand-Shilov spaces and their duals.

Alden Waters

University of Leeds

TITLE: **Analytic Properties of Heat Equation Solutions and Reachable Sets.**

We consider heat equations on bounded Lipschitz domains Omega in R^d and show that solutions to the heat equation for positive times are analytically extendable to a subdomain of the complex plane containing Omega. Our analysis is based on the boundary layer potential method for the heat equation. In particular, our method gives an explanation for the shapes appearing in the literature in 1 dimension, which is not so easy to explain using Fourier analysis alone. I will also discuss the converse theorem, namely that certain sets in the complex plane can be realized as solutions to the heat equation on the boundary of a ball Ω. Boundary layer potential theory also gives an indication that this statement is more difficult if *Omega* is not a ball. This exciting new technique to analyze the question of reachable sets is joint work with Alexander Strohmaier.